1. D $y' = x^2 + 10x$; $y'' = 2x + 10$; y'' changes sign at $x = -5$

2. B $\int_{-1}^{4} f(x)dx = \int_{-1}^{2} f(x)dx + \int_{2}^{4} f(x)dx$

$= \text{Area of trapezoid(1)} - \text{Area of trapezoid(2)} = 4 - 1.5 = 2.5$

3. C $\int_1^2 \frac{1}{x^2}dx = \int_1^2 x^{-2}dx = -x^{-1}\Big|_1^2 = \frac{1}{2}$

4. B This would be false if f was a linear function with non-zero slope.

5. E $\int_0^x \sin t\, dt = -\cos t\Big|_0^x = -\cos x - (-\cos 0) = -\cos x + 1 = 1 - \cos x$

6. A Substitute $x = 2$ into the equation to find $y = 3$. Taking the derivative implicitly gives $\frac{d}{dx}\left(x^2 + xy\right) = 2x + xy' + y = 0$. Substitute for x and y and solve for y'.

$4 + 2y' + 3 = 0;\ y' = -\frac{7}{2}$

7. E $\int_1^e \frac{x^2-1}{x}dx = \int_1^e x - \frac{1}{x}dx = \left(\frac{1}{2}x^2 - \ln x\right)\Big|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

8. E $h(x) = f(x)g(x)$ so, $h'(x) = f'(x)g(x) + f(x)g'(x)$. It is given that $h'(x) = f(x)g'(x)$. Thus, $f'(x)g(x) = 0$. Since $g(x) > 0$ for all x, $f'(x) = 0$. This means that f is constant. It is given that $f(0) = 1$, therefore $f(x) = 1$.

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about 3,000 barrels.

10. D $f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2}$; $f'(2) = \frac{(2-1)(4) - (4-2)(1)}{(2-1)^2} = 2$

11. A Since f is linear, its second derivative is zero. The integral gives the area of a rectangle with zero height and width $(b-a)$. This area is zero.

12. E $\lim_{x\to 2^-} f(x) = \ln 2 \neq 4\ln 2 = \lim_{x\to 2^+} f(x)$. Therefore the limit does not exist.

13. B At $x = 0$ and $x = 2$ only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other x's.

14. C $v(t) = 2t - 6$; $v(t) = 0$ for $t = 3$

15 D By the Fundamental Theorem of Calculus, $F'(x) = \sqrt{x^3+1}$, thus $F'(2) = \sqrt{2^3+1} = \sqrt{9} = 3$.

16. E $f'(x) = \cos(e^{-x})\cdot\frac{d}{dx}(e^{-x}) = \cos(e^{-x})\left(e^{-x}\cdot\frac{d}{dx}(-x)\right) = -e^{-x}\cos(e^{-x})$

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x = 1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x = 1$, $f''(1) < 0$.

18. B $y' = 1 - \sin x$ so $y'(0) = 1$ and the line with slope 1 containing the point (0,1) is $y = x + 1$.

19. C Points of inflection occur where f'' changes sign. This is only at $x = 0$ and $x = -1$. There is no sign change at $x = 2$.

20. A $\int_{-3}^{k} x^2\,dx = \frac{1}{3}x^3\Big|_{-3}^{k} = \frac{1}{3}\left(k^3 - (-3)^3\right) = \frac{1}{3}\left(k^3 + 27\right) = 0$ only when $k = -3$.

21. B The solution to this differential equation is known to be of the form $y = y(0)\cdot e^{kt}$. Option (B) is the only one of this form. If you do not remember the form of the solution, then separate the variables and antidifferentiate.

$$\frac{dy}{y} = k\,dt;\ \ln|y| = kt + c_1;\ |y| = e^{kt+c_1} = e^{kt}e^{c_1};\ y = ce^{kt}.$$

22. C f is increasing on any interval where $f'(x) > 0$. $f'(x) = 4x^3 + 2x = 2x(2x^2+1) > 0$. Since $(x^2+1) > 0$ for all x, $f'(x) > 0$ whenever $x > 0$.

23. A The graph shows that f is increasing on an interval (a,c) and decreasing on the interval (c,b), where $a < c < b$. This means the graph of the derivative of f is positive on the interval (a,c) and negative on the interval (c,b), so the answer is (A) or (E). The derivative is not (E), however, since then the graph of f would be concave down for the entire interval.

24. D The maximum acceleration will occur when its derivative changes from positive to negative or at an endpoint of the interval. $a(t) = v'(t) = 3t^2 - 6t + 12 = 3(t^2 - 2t + 4)$ which is always positive. Thus the acceleration is always increasing. The maximum must occur at $t = 3$ where $a(3) = 21$

25. D The area is given by $\int_0^2 x^2 - (-x)\,dx = \left(\frac{1}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$.

26. A Any value of k less than 1/2 will require the function to assume the value of 1/2 at least twice because of the Intermediate Value Theorem on the intervals $[0,1]$ and $[1,2]$. Hence $k = 0$ is the only option.

27. A $\frac{1}{2}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{2}\int_0^2 (x^3+1)^{\frac{1}{2}}\left(\frac{1}{3}\cdot 3x^2\right)dx = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{\frac{3}{2}}\Big|_0^2 = \frac{1}{9}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{26}{9}$

28. E $f'(x) = \sec^2(2x)\cdot\frac{d}{dx}(2x) = 2\sec^2(2x)$; $f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right) = 2(4) = 8$

76. A From the graph it is clear that f is not continuous at $x = a$. All others are true.

77. C Parallel tangents will occur when the slopes of f and g are equal. $f'(x) = 6e^{2x}$ and $g'(x) = 18x^2$. The graphs of these derivatives reveal that they are equal only at $x = -0.391$.

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

79. A The graph of the derivative would have to change from positive to negative. This is only true for the graph of f'.

80. B Look at the graph of $f'(x)$ on the interval $(0,10)$ and count the number of x-intercepts in the interval.

81. D Only II is false since the graph of the absolute value function has a sharp corner at $x = 0$.

82. E Since F is an antiderivative of f, $\int_1^3 f(2x)\,dx = \frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}\left(F(6) - F(2)\right)$

83. B $\lim_{x\to a}\frac{x^2 - a^2}{x^4 - a^4} = \lim_{x\to a}\frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} = \lim_{x\to a}\frac{1}{(x^2 + a^2)} = \frac{1}{2a^2}$

84. A A known solution to this differential equation is $y(t) = y(0)e^{kt}$. Use the fact that the population is $2y(0)$ when $t = 10$. Then
$2y(0) = y(0)e^{k(10)} \Rightarrow e^{10k} = 2 \Rightarrow k = (0.1)\ln 2 = 0.069$

85. C There are 3 trapezoids. $\frac{1}{2}\cdot 3\left(f(2) + f(5)\right) + \frac{1}{2}\cdot 2\left(f(5) + f(7)\right) + \frac{1}{2}\cdot 1\left(f(7) + f(8)\right)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by
$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8\left(\frac{8 - x}{2}\right)^2 dx = 16.755$$

87. D Find the x for which $f'(x) = 1$. $f'(x) = 4x^3 + 4x = 1$ only for $x = 0.237$. Then $f(0.237) = 0.115$. So the equation is $y - 0.115 = x - 0.237$. This is equivalent to option (D).

88. C $F(9)-F(1)=\int_1^9 \frac{(\ln t)^3}{t}\,dt = 5.827$ using a calculator. Since $F(1)=0$, $F(9)=5.827$.

Or solve the differential equation with an initial condition by finding an antiderivative for $\frac{(\ln x)^3}{x}$. This is of the form u^3du where $u=\ln x$. Hence $F(x)=\frac{1}{4}(\ln x)^4+C$ and since $F(1)=0$, $C=0$. Therefore $F(9)=\frac{1}{4}(\ln 9)^4=5.827$

89. B The graph of $y=x^2-4$ is a parabola that changes from positive to negative at $x=-2$ and from negative to positive at $x=2$. Since g is always negative, f' changes sign opposite to the way $y=x^2-4$ does. Thus f has a relative minimum at $x=-2$ and a relative maximum at $x=2$.

90. D The area of a triangle is given by $A=\frac{1}{2}bh$. Taking the derivative with respect to t of both sides of the equation yields $\frac{dA}{dt}=\frac{1}{2}\left(\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}\right)$. Substitute the given rates to get $\frac{dA}{dt}=\frac{1}{2}(3h-3b)=\frac{3}{2}(h-b)$. The area will be decreasing whenever $\frac{dA}{dt}<0$. This is true whenever $b>h$.

91. E I. True. Apply the Intermediate Value Theorem to each of the intervals $[2,5]$ and $[5,9]$.

II. True. Apply the Mean Value Theorem to the interval $[2,9]$.

III. True. Apply the Intermediate Value Theorem to the interval $[2,5]$.

92. D $\int_k^{\frac{\pi}{2}} \cos x\,dx = 0.1 \Rightarrow \sin\left(\frac{\pi}{2}\right)-\sin k = 0.1 \Rightarrow \sin k = 0.9$. Therefore $k=\sin^{-1}(0.9)=1.120$.