1. C f will be increasing when its derivative is positive.
$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$ $f'(x) = 3(x+3)(x-1) > 0$ for $x < -3$ or $x > 1$.

2. A $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = 3 \Rightarrow \frac{dy}{dx} = \frac{3}{5}$

3. D Find the derivative implicitly and substitute. $2y \cdot y' + 3(xy+1)^2(x \cdot y' + y) = 0$;
$2(-1) \cdot y' + 3((2)(-1)+1)^2((2) \cdot y' + (-1)) = 0$; $-2y' + 6 \cdot y' - 3 = 0$; $y' = \frac{3}{4}$

4. A Use partial fractions. $\frac{1}{x^2 - 6x + 8} = \frac{1}{(x-4)(x-2)} = \frac{1}{2}\left(\frac{1}{x-4} - \frac{1}{x-2}\right)$

$$\int \frac{1}{x^2 - 6x + 8}\,dx = \frac{1}{2}\left(\ln|x-4| - \ln|x-2|\right) + C = \frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$$

5. A $h'(x) = f'(g(x)) \cdot g'(x)$; $h''(x) = f''(g(x)) \cdot g'(x) \cdot g'(x) + f'(g(x)) \cdot g''(x)$
$h''(x) = f''(g(x)) \cdot (g'(x))^2 + f'(g(x)) \cdot g''(x)$

6. E The graph of h has 2 turning points and one point of inflection. The graph of h' will have 2 x-intercepts and one turning point. Only (C) and (E) are possible answers. Since the first turning point on the graph of h is a relative maximum, the first zero of h' must be a place where the sign changes from positive to negative. This is option (E).

7. E $\int_1^e \frac{x^2 - 1}{x}\,dx = \int_1^e x - \frac{1}{x}\,dx = \left(\frac{1}{2}x^2 - \ln x\right)\Big|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

8. B $y(x) = -\frac{1}{3}(\cos x)^3 + C$; Let $x = \frac{\pi}{2}$, $0 = -\frac{1}{3}\left(\cos\frac{\pi}{2}\right)^3 + C \Rightarrow C = 0$. $y(0) = -\frac{1}{3}(\cos 0)^3 = -\frac{1}{3}$

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about 3,000 barrels.

10. E $v(t) = \left(3t^2 - 1, 6(2t-1)^2\right)$ and $a(t) = \left(6t, 24(2t-1)\right) \Rightarrow a(1) = (6, 24)$

11. A Since f is linear, its second derivative is zero and the integral gives the area of a rectangle with zero height and width $(b-a)$. This area is zero.

12. E $\lim_{x\to 2^-} f(x) = \ln 2 \neq 4\ln 2 = \lim_{x\to 2^+} f(x)$. Therefore the limit does not exist.

13. B At $x=0$ and $x=2$ only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other x's.

14. E $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$; $\sin 1 \approx 1 - \frac{1^3}{3!} + \frac{1^5}{5!} = 1 - \frac{1}{6} + \frac{1}{120}$

15. B Use the technique of antiderivatives by parts. Let $u = x$ and $dv = \cos x\, dx$.

$$\int x\cos x\, dx = x\sin x - \int \sin x\, dx = x\sin x + \cos x + C$$

16. C Inflection point will occur when f'' changes sign. $f'(x) = 15x^4 - 20x^3$.
$f''(x) = 60x^3 - 60x^2 = 60x^2(x-1)$. The only sign change is at $x = 1$.

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x=1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x=1$, $f''(1) < 0$.

18. B
I. Divergent. The limit of the nth term is not zero.
II. Convergent. This is the same as the alternating harmonic series.
III. Divergent. This is the harmonic series.

19. D Find the points of intersection of the two curves to determine the limits of integration.
$4\sin\theta = 2$ when $\sin\theta = 0.5$; this is at $\theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$. Area $= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left((4\sin\theta)^2 - (2)^2\right)d\theta$

20. E $\left.\frac{d\left(\sqrt[3]{x}\right)}{dt}\right|_{x=8} = \left.\frac{1}{3}x^{-\frac{2}{3}}\cdot\frac{dx}{dt}\right|_{x=8} = \frac{1}{3}(8)^{-\frac{2}{3}}\cdot\frac{dx}{dt} = \frac{1}{3}\cdot\frac{1}{4}\cdot\frac{dx}{dt} = \frac{1}{12}\cdot\frac{dx}{dt} \Rightarrow k = 12$

21 C The length of this parametric curve is given by $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{(t^2)^2 + t^2}\,dt$.

22. A This is the integral test applied to the series in (A). Thus the series in (A) converges. None of the others must be true.

23. E I. False. The relative maximum could be at a cusp.
II. True. There is a critical point at $x = c$ where $f'(c)$ exists
III. True. If $f''(c) > 0$, then there would be a relative minimum, not maximum

24. C All slopes along the diagonal $y = -x$ appear to be 0. This is consistent only with option (C). For each of the others you can see why they do not work. Option (A) does not work because all slopes at points with the same x coordinate would have to be equal. Option (B) does not work because all slopes would have to be positive. Option (D) does not work because all slopes in the third quadrant would have to be positive. Option (E) does not work because there would only be slopes for $y > 0$.

25. C $\int_0^\infty x^2 e^{-x^3}\,dx = \lim_{b\to\infty}\int_0^b x^2 e^{-x^3}\,dx = \lim_{b\to\infty} -\frac{1}{3}e^{-x^3}\Big|_0^b = \frac{1}{3}.$

26. E As $\lim_{t\to\infty}\frac{dP}{dt} = 0$ for a population satisfying a logistic differential equation, this means that $P \to 10{,}000$.

27. D If $f(x) = \sum_{n=0}^{\infty} a_n x^n$, then $f'(x) = \sum_{n=0}^{\infty} n a_n x^{n-1} = \sum_{n=1}^{\infty} n a_n x^{n-1}$.

$f'(1) = \sum_{n=1}^{\infty} n a_n 1^{n-1} = \sum_{n=1}^{\infty} n a_n$

28. C Apply L'Hôpital's rule. $\lim_{x\to 1}\frac{\int_1^x e^{t^2}\,dt}{x^2 - 1} = \lim_{x\to 1}\frac{e^{x^2}}{2x} = \frac{e}{2}$

76. D The first series is either the harmonic series or the alternating harmonic series depending on whether k is odd or even. It will converge if k is odd. The second series is geometric and will converge if $k < 4$.

77. E $f'(t) = \left(-e^{-t}, -\sin t\right)$; $f''(t) = \left(e^{-t}, -\cos t\right)$.

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

79. A None. For every positive value of a the denominator will be zero for some value of x.

80. B The area is given by $\int_{-\frac{2}{3}}^{\frac{2}{3}} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$

81. B $\frac{dy}{dx} = \sqrt{1-y^2}$; $\frac{d^2y}{dx^2} = \frac{d}{dx}\left((1-y^2)^{\frac{1}{2}}\right) = \frac{1}{2}\left(1-y^2\right)^{-\frac{1}{2}} \cdot (-2y) \cdot \frac{dy}{dx} = -y$

82. B $\int_3^5 [f(x)+g(x)]\,dx = \int_3^5 [2g(x)+7]\,dx = 2\int_3^5 g(x)\,dx + (7)(2) = 2\int_3^5 g(x)\,dx + 14$

83. C Use a calculator. The maximum for $\left|\ln x - \left(\frac{(x-1)}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}\right)\right|$ on the interval $0.3 \le x \le 1.7$ occurs at $x = 0.3$.

84. B You may use the ratio test. However, the series will converge if the numerator is $(-1)^n$ and diverge if the numerator is 1^n. Any value of x for which $|x+2| > 1$ in the numerator will make the series diverge. Hence the interval is $-3 \le x < -1$.

85. C There are 3 trapezoids. $\frac{1}{2} \cdot 3\left(f(2) + f(5)\right) + \frac{1}{2} \cdot 2\left(f(5) + f(7)\right) + \frac{1}{2} \cdot 1\left(f(7) + f(8)\right)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by

$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$$

87. D Find the x for which $f'(x)=1$. $f'(x)=4x^3+4x=1$ only for $x=0.237$. Then $f(0.237)=0.115$. So the equation is $y-0.115=x-0.237$. This is equivalent to option (D).

88. C From the given information, f is the derivative of g. We want a graph for f that represents the slopes of the graph g. The slope of g is zero at a and b. Also the slope of g changes from positive to negative at one point between a and b. This is true only for figure (C).

89. A The series is the Maclaurin expansion of e^{-x}. Use the calculator to solve $e^{-x} \doteq x^3$.

90. A Constant acceleration means linear velocity which in turn leads to quadratic position. Only the graph in (A) is quadratic with initial $s=2$.

91. E $v(t)=11+\int_0^t a(x)\,dx \approx 11+[2\cdot 5+2\cdot 2+2\cdot 8]=41$ ft/sec.

92. D $f'(x)=2x-2$, $f'(2)=2$, and $f(2)=3$, so an equation for the tangent line is $y=2x-1$. The difference between the function and the tangent line is represented by $(x-2)^2$. Solve $(x-2)^2<0.5$. This inequality is satisfied for all x such that $2-\sqrt{0.5}<x<2+\sqrt{0.5}$. This is the same as $1.293<x<2.707$. Thus the largest value in the list that satisfies the inequality is 2.7.