

AP Calculus Brief Review

* means topic only on BC

<p>Curve sketching and analysis $y = f(x)$ must be continuous at each: critical point: $\frac{dy}{dx} = 0$ or <u>undefined</u> local minimum: or <u>endpoints</u> $\frac{dy}{dx}$ goes $(-,0,+)$ or $(-,und,+)$ or $\frac{d^2y}{dx^2} > 0$ local maximum: $\frac{dy}{dx}$ goes $(+,0,-)$ or $(+,und,-)$ or $\frac{d^2y}{dx^2} < 0$ point of inflection: concavity changes $\frac{d^2y}{dx^2}$ goes from $(+,0,-)$, $(-,0,+)$, $(+,und,-)$, or $(-,und,+)$</p>	<p>Differentiation Rules Chain Rule $\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$ OR $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ Product Rule $\frac{d}{dx}(uv) = \frac{du}{dx}v + u \frac{dv}{dx}$ OR $u'v + uv'$ Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u \frac{dv}{dx}}{v^2}$ OR $\frac{u'v - uv'}{v^2}$</p>	<p>Approx. Methods for Integration Trapezoidal Rule $\int_a^b f(x)dx = \frac{1}{2} \frac{b-a}{n} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$ Simpson's Rule $\int_a^b f(x)dx = \frac{1}{3} \Delta x [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$</p>
<p>Basic Derivatives $\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$ $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$ $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$ where u is a function of x, and a is a constant.</p>	<p>“PLUS A CONSTANT” The Fundamental Theorem of Calculus $\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$</p>	<p>Theorem of the Mean Value i.e. AVERAGE VALUE If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that $f(c) = \frac{\int_a^b f(x)dx}{(b-a)}$ This value $f(c)$ is the “average value” of the function on the interval $[a, b]$.</p>
<p>More Derivatives $\frac{d}{dx}\left(\sin^{-1} \frac{u}{a}\right) = \frac{1}{\sqrt{a^2 - u^2}} \frac{du}{dx}$ $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\tan^{-1} \frac{u}{a}\right) = \frac{a}{a^2 + u^2} \cdot \frac{du}{dx}$ $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$ $\frac{d}{dx}\left(\sec^{-1} \frac{u}{a}\right) = \frac{a}{ u \sqrt{u^2 - a^2}} \cdot \frac{du}{dx}$ $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{ x \sqrt{x^2 - 1}}$ $\frac{d}{dx}(a^{u(x)}) = a^{u(x)} \ln a \cdot \frac{du}{dx}$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$</p>	<p>Corollary to FTC $\frac{d}{dx} \int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(a(x))a'(x)$</p>	<p>Solids of Revolution and friends Disk Method $V = \pi \int_{x=a}^{x=b} [R(x)]^2 dx$ Washer Method $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$ General volume equation (not rotated) $V = \int_a^b Area(x) dx$ *Arc Length $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ $= \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$</p>
<p>Derivative of Inverse Functions If $y = f(x)$ and $x = f^{-1}(x)$ are differentiable inverse functions, then their derivatives are inverses. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$</p>	<p>Intermediate Value Theorem If the function $f(x)$ is continuous on $[a, b]$, and y is a number between $f(a)$ and $f(b)$, then there exists at least one number $x = c$ in the open interval (a, b) such that $f(c) = y$.</p>	<p>Distance, Velocity, and Acceleration velocity = $\frac{d}{dt}$ (position) acceleration = $\frac{d}{dt}$ (velocity) *velocity vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$ speed = $v = \sqrt{(x')^2 + (y')^2}$ * displacement = $\int_{t_0}^{t_f} v dt$ distance = $\int_{\text{initial time}}^{\text{final time}} v dt$ $\int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2} dt$ * average velocity = $= \frac{\text{final position} - \text{initial position}}{\text{total time}}$ $= \frac{\Delta x}{\Delta t}$</p>
<p>Mean Value Theorem If the function $f(x)$ is continuous on $[a, b]$, AND the first derivative exists on the interval (a, b), then there is at least one number $x = c$ in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$</p>	<p>Derivative of Inverse Functions If $y = f(x)$ and $x = f^{-1}(x)$ are differentiable inverse functions, then their derivatives are inverses. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$</p>	<p>Distance, Velocity, and Acceleration velocity = $\frac{d}{dt}$ (position) acceleration = $\frac{d}{dt}$ (velocity) *velocity vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$ speed = $v = \sqrt{(x')^2 + (y')^2}$ * displacement = $\int_{t_0}^{t_f} v dt$ distance = $\int_{\text{initial time}}^{\text{final time}} v dt$ $\int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2} dt$ * average velocity = $= \frac{\text{final position} - \text{initial position}}{\text{total time}}$ $= \frac{\Delta x}{\Delta t}$</p>

BC TOPICS and important TRIG identities and values

<p>L'Hôpital's Rule If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$</p>	<p>Slope of a Parametric equation Given a $x(t)$ and a $y(t)$ the slope is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$</p>	<p>Values of Trigonometric Functions for Common Angles</p> <table border="1" data-bbox="1096 189 1526 630"> <thead> <tr> <th>θ</th> <th>$\sin \theta$</th> <th>$\cos \theta$</th> <th>$\tan \theta$</th> </tr> </thead> <tbody> <tr> <td>0°</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>$\frac{\pi}{6}$</td> <td>$\frac{1}{2}$</td> <td>$\frac{\sqrt{3}}{2}$</td> <td>$\frac{\sqrt{3}}{3}$</td> </tr> <tr> <td>$\frac{\pi}{4}$</td> <td>$\frac{\sqrt{2}}{2}$</td> <td>$\frac{\sqrt{2}}{2}$</td> <td>1</td> </tr> <tr> <td>$\frac{\pi}{3}$</td> <td>$\frac{\sqrt{3}}{2}$</td> <td>$\frac{1}{2}$</td> <td>$\sqrt{3}$</td> </tr> <tr> <td>$\frac{\pi}{2}$</td> <td>1</td> <td>0</td> <td>"∞"</td> </tr> <tr> <td>π</td> <td>0</td> <td>-1</td> <td>0</td> </tr> </tbody> </table> <p>Know both the <i>inverse trig</i> and the <i>trig values</i>. E.g. $\tan(\pi/4)=1$ & $\tan^{-1}(1)=\pi/4$</p>	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	0°	0	1	0	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\pi}{2}$	1	0	" ∞ "	π	0	-1	0
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<p>Euler's Method If given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through (x_0, y_0), $y(x_0) = y_0$ \vdots $y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot \Delta x$ In other words: $x_{\text{new}} = x_{\text{old}} + \Delta x$ $y_{\text{new}} = y_{\text{old}} + \left. \frac{dy}{dx} \right _{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$</p>	<p>Polar Curve For a polar curve $r(\theta)$, the AREA inside a "leaf" is $\int_{\theta_1}^{\theta_2} \frac{1}{2} [r(\theta)]^2 d\theta$ where θ_1 and θ_2 are the "first" two times that $r = 0$. The SLOPE of $r(\theta)$ at a given θ is $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta} [r(\theta) \sin \theta]}{\frac{d}{d\theta} [r(\theta) \cos \theta]}$</p>	<p>Trig Identities <i>Double Argument</i> $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ <i>Pythagorean</i> $\sin^2 x + \cos^2 x = 1$ (others are easily derivable by dividing by $\sin^2 x$ or $\cos^2 x$) $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$ <i>Reciprocal</i> $\sec x = \frac{1}{\cos x}$ or $\cos x \sec x = 1$ $\csc x = \frac{1}{\sin x}$ or $\sin x \csc x = 1$ <i>Odd-Even</i> $\sin(-x) = -\sin x$ (odd) $\cos(-x) = \cos x$ (even) <i>Some more handy INTEGRALS:</i> $\int \tan x dx = \ln \sec x + C$ $= -\ln \cos x + C$ $\int \sec x dx = \ln \sec x + \tan x + C$</p>																												
<p>Integration by Parts $\int u dv = uv - \int v du$</p> <p>Integral of Log Use IBP and let $u = \ln x$ (Recall $u=LIPET$) $\int \ln x dx = x \ln x - x + C$</p>	<p>Ratio Test The series $\sum_{k=0}^{\infty} a_k$ converges if $\lim_{k \rightarrow \infty} \left \frac{a_{k+1}}{a_k} \right < 1$ If the limit equal 1, you know nothing.</p>	<p>Maclaurin Series A Taylor Series about $x = 0$ is called Maclaurin. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ $\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$</p>																												
<p>Taylor Series If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial $f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$</p>	<p>Lagrange Error Bound If $P_n(x)$ is the n^{th} degree Taylor polynomial of $f(x)$ about c and $f^{(n+1)}(t) \leq M$ for all t between x and c, then $f(x) - P_n(x) \leq \frac{M}{(n+1)!} x-c ^{n+1}$</p>	<p>Alternating Series Error Bound If $S_N = \sum_{k=1}^N (-1)^k a_n$ is the N^{th} partial sum of a convergent alternating series, then $S_{\infty} - S_N \leq a_{N+1}$</p> <p>Geometric Series $a + ar + ar^2 + ar^3 + \dots + ar^{n-1} + \dots = \sum_{n=1}^{\infty} ar^{n-1}$ diverges if $r \geq 1$; converges to $\frac{a}{1-r}$ if $r < 1$</p>																												
<p>Maclaurin Series A Taylor Series about $x = 0$ is called Maclaurin. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ $\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$</p>	<p>Exponential Functions "Rate of change directly proportional to quantity" $\frac{dy}{dt} = ky$ $y = y_0 e^{kt}$</p>	<p>Newton's Method of finding roots $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$</p>																												
<p>Logarithmic Functions C is carrying capacity. $\frac{dP}{dt} = kP(C - P)$ or $P(t) = \frac{C}{1 + de^{-Ckt}}$</p>																														